Principles of Communications
ECS 332

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Summary

Office Hours:

BKD, 6th floor of Sirindhralai building

Wednesday	**14:00-15:30**
Friday	**14:00-15:30**

Fourier Series

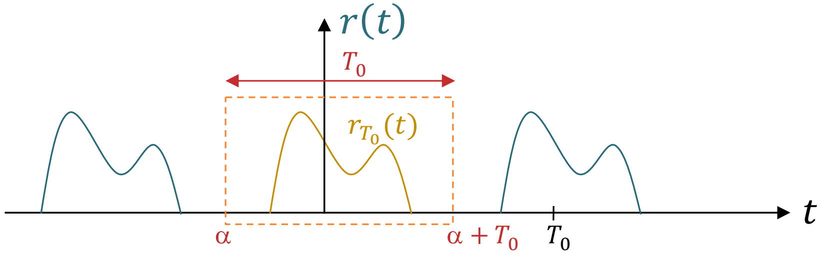
A **periodic** signal $r(t)$ with period T_0 can be "expanded" into a linear combination of complex-exponential functions:

$$r(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi(k f_0)t}$$

$$f_0 = \frac{1}{T_0}$$

Fourier coefficients: $c_k = \frac{1}{T_0} R_{T_0}(k f_0) = \langle r(t) e^{-j2\pi(k f_0)t} \rangle$

Special Case:
$c_0 = \langle r(t) \rangle$



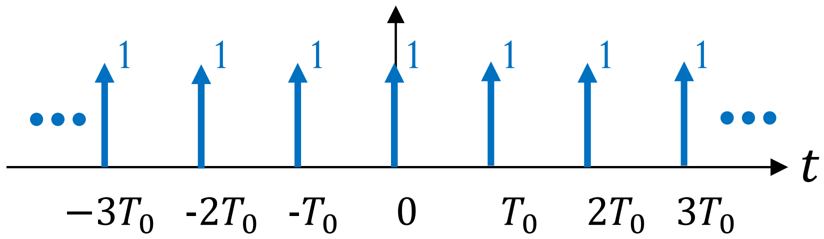
$r_{T_0}(t) \xrightarrow{\mathcal{F}} R_{T_0}(f)$

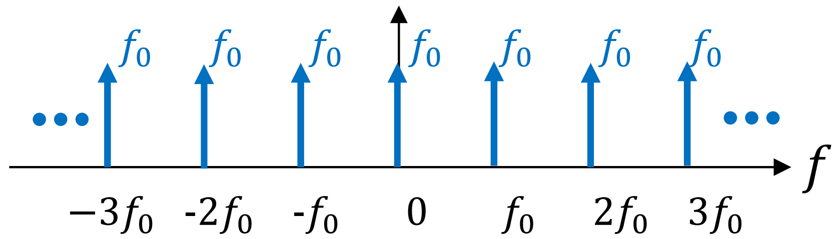
One period of $r(t)$

$$P_r = \sum_{k=-\infty}^{\infty} |c_k|^2$$

Important Fourier Transform Pair

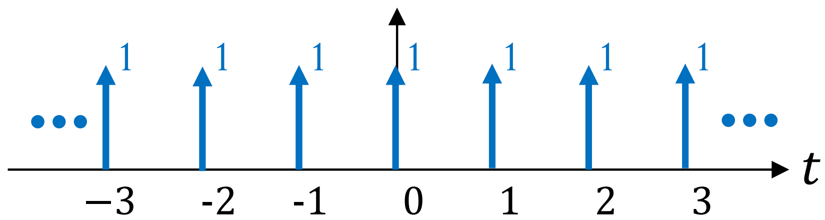
$$\sum_{n=-\infty}^{\infty} \delta\left(t - nT_0\right) = \sum_{k=-\infty}^{\infty} \frac{1}{T_0} e^{j2\pi(kf_0)t} \quad \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} \quad f_0 \sum_{k=-\infty}^{\infty} \delta\left(t - kf_0\right)$$

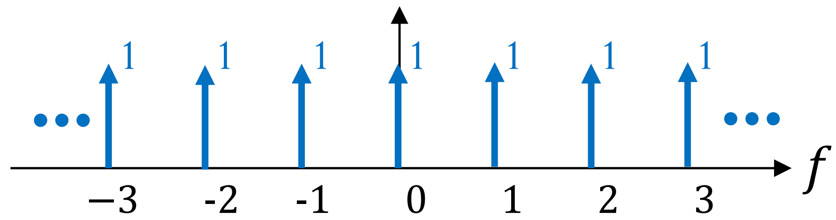




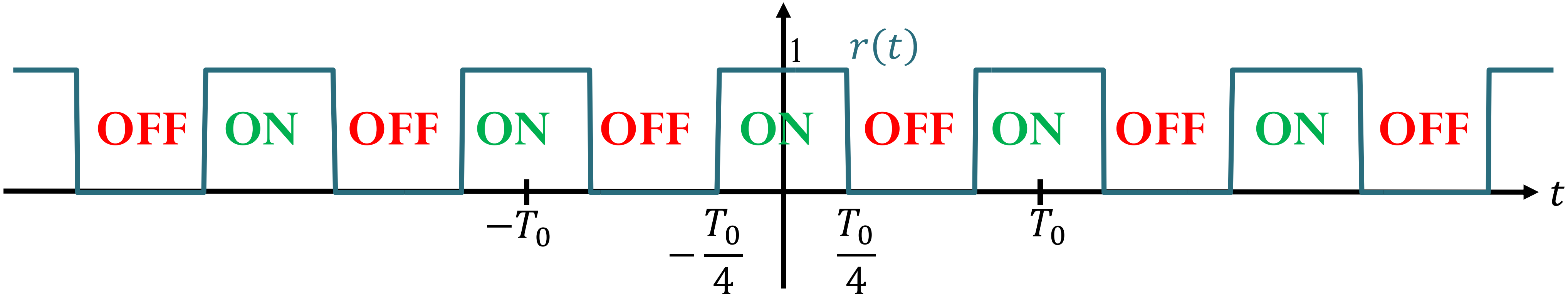
In particular,

$$\sum_{n=-\infty}^{\infty} \delta\left(t - n\right) \quad \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} \quad \sum_{k=-\infty}^{\infty} \delta\left(f - k\right)$$

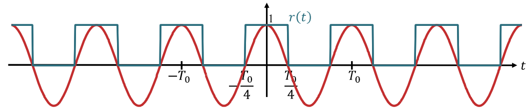




Square pulse periodic signal



$$r(t) = 1[\cos(2\pi f_0 t) \geq 0] = \begin{cases} 1, & \cos(2\pi f_0 t) \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$



Fourier series expansion:

$$r(t) = \frac{1}{2} + \frac{1}{\pi} e^{j(2\pi f_0 t)} - \frac{1}{3\pi} e^{j(2\pi(3f_0)t)} + \frac{1}{5\pi} e^{j(2\pi(5f_0)t)} + \cdots$$
$$+ \frac{1}{\pi} e^{j(2\pi(-f_0)t)} - \frac{1}{3\pi} e^{j(2\pi(-3f_0)t)} + \frac{1}{5\pi} e^{j(2\pi(-5f_0)t)} + \cdots$$

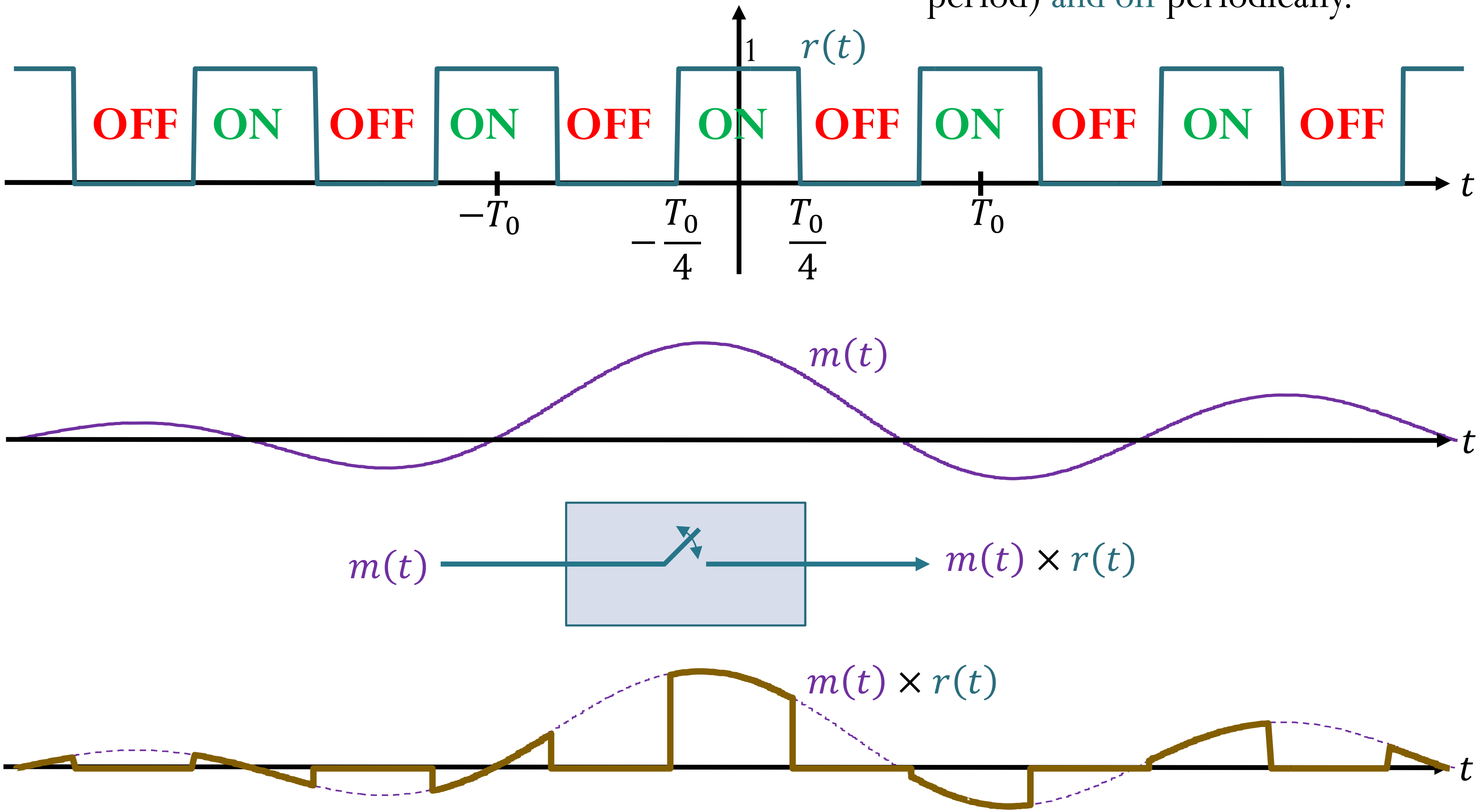
Trigonometric Fourier series expansion:

$$e^{jx} + e^{-jx} = 2\cos(x)$$

$$r(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_0 t) - \frac{2}{3\pi}\cos(2\pi(3f_0)t) + \frac{2}{5\pi}\cos(2\pi(5f_0)t) + \cdots$$

Switching Operation

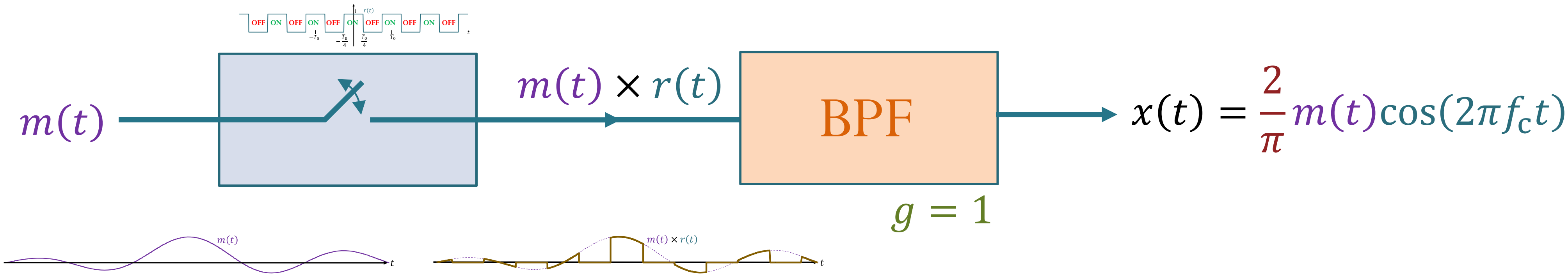
Multiplying a signal $m(t)$ by the square-wave $r(t)$ is equivalent to switching $m(t)$ on (for half a period) and off periodically.



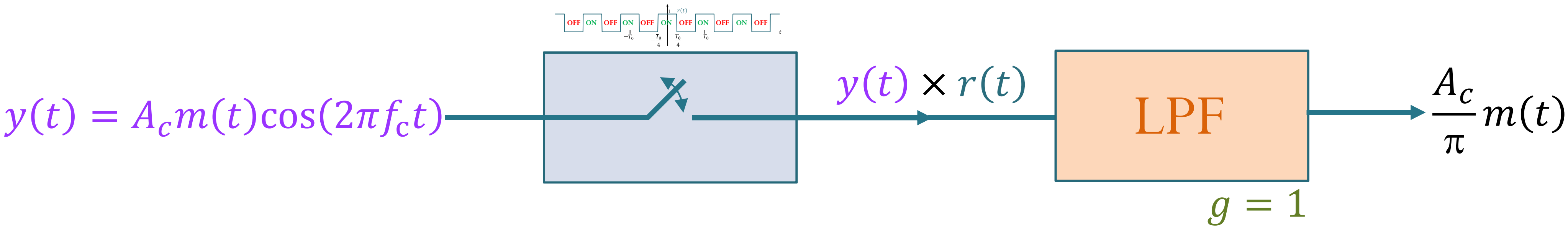
Switching Modem

Multiplying a signal $m(t)$ by the square-wave $r(t)$ is equivalent to switching $m(t)$ on (for half a period) and off periodically.

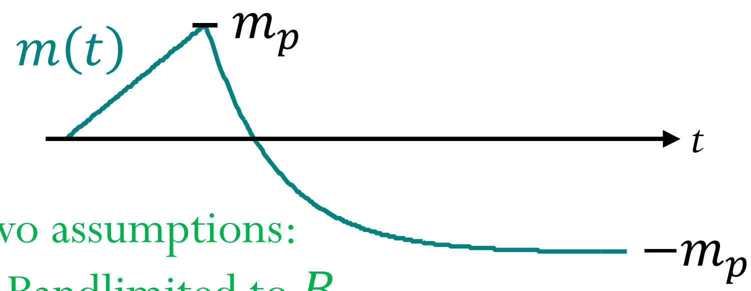
Switching Modulator:



Switching Demodulator:



AM Modulation



Two assumptions:

1. Bandlimited to B
2. Bounded between $\pm m_p$

$$x_{\mathrm{AM}}(t) = \big(A + m(t)\big)\cos(2\pi f_c t) = A\cos(2\pi f_c t) + m(t)\cos(2\pi f_c t)$$

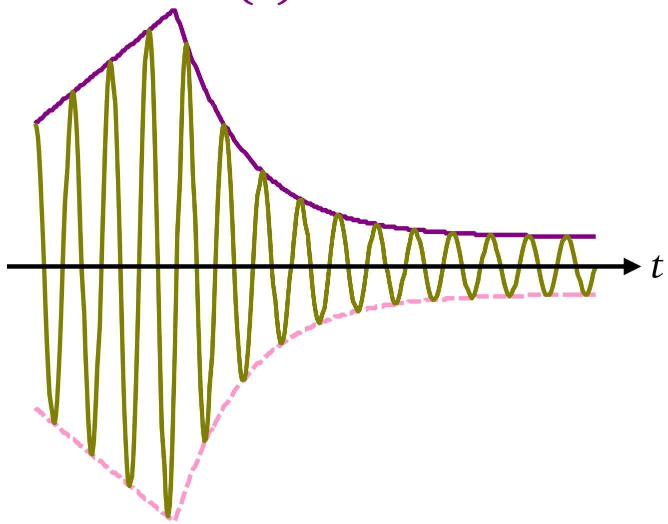
Modulation index: $\mu = \frac{m_p}{A}$

Power Efficiency $= \frac{P_m}{A^2 + P_m} = \frac{1}{1 + \frac{m_p^2}{\mu^2 P_m}} \leq 50\%$

$\frac{m_p^2}{\mu^2 P_m} \geq 1$

$\mu \leq 100\%$

$A + m(t) \geq 0 \quad$ for all t

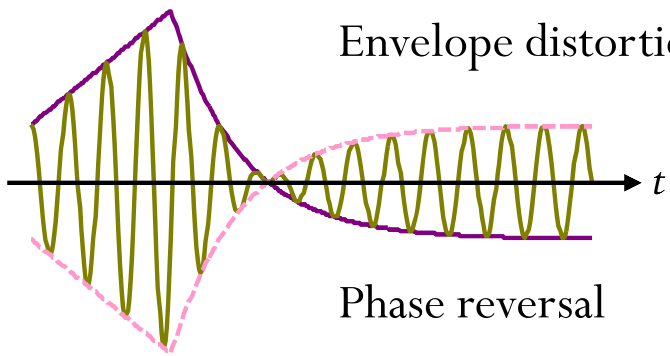


$\mu > 100\%$

$A + m(t) < 0 \quad$ for some t

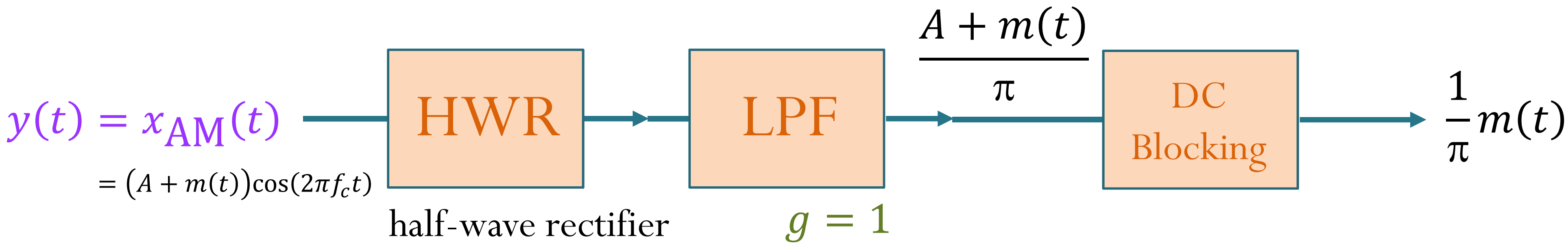
Overmodulation

Envelope distortion



Phase reversal

Rectifier Demodulator for AM



Instantaneous Frequency

- Sinusoidal signal:

$$g(t) = A\cos(2\pi \boldsymbol{f_0} t + \phi)$$

  - Frequency = $\boldsymbol{f_0}$

- Generalized sinusoidal signal:

$$g(t) = A\cos\big(\theta(t)\big)$$

  - The **instantaneous frequency** at time *t* is given by

$$\boldsymbol{f(t)} = \frac{1}{2\pi}\frac{d}{dt}\theta(t)$$

Angle Modulation

- **Phase Modulation**:

$$x_{\mathrm{PM}}(t) = A\cos\left(\underbrace{2\pi f_c t + \phi + k_p m(t)}\right)$$

$$\Big\Downarrow \frac{d}{dt}$$

  - Instantaneous Frequency: $f(t) = f_c + k_p \frac{d}{dt} m(t)$

- **Frequency Modulation**:

$$x_{\mathrm{FM}}(t) = A\cos\left(\underbrace{2\pi f_c t + \phi + 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau}\right)$$

$$\Big\Downarrow \frac{d}{dt}$$

  - Instantaneous Frequency: $f(t) = f_c + k_f m(t)$